



Reasoning: Reductio Ad Absurdum

By
Ronald P. Menich © 2007

**New at FORum:
Children's
program from
noon to 3 p.m.**

Individual Highlights

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Twenty-five centuries ago a solemn group of Greek mathematicians and philosophers rowed a boat directly away from shore and headed out into the wine-dark sea. It was sunset on a beautiful summer evening. The last slanting rays of sun left the bottom of the boat in shadow and in that shadow laid another mathematician, this one bound with cord and blindfolded. The others rowed in silence and when their eyes occasionally came to rest on the bound man, they hastily looked away with revulsion.

The sun set behind the mountains to the rear of the boat, and after some time the rowers stopped. One of the men, apparently the leader, rose to stand in the gathering gloom and addressed the prisoner, looking sternly down at him from above.

"Hippasus of Metapontum, do you renounce the heretical notions with which you have attempted to poison us? How could you so betray your fellow scholars?"

Hippasus replied, "I do not and I cannot renounce the truth, Pythagoras you idiot! Mathematics does not lie! Your mother swims after Persian troop ships!"

Pythagoras motioned angrily to the other men in the boat. They quickly and roughly grabbed Hippasus and tossed him overboard, after which they then also tossed into the sea a large rock around which a rope was tied, the other end of which was attached to the waist of the unfortunate Hippasus. Down, down, down sank Hippasus and as he sank he could be heard blubbling, "The square root of 2 is irrational!"

Well, the Greeks of the Classical period were not perfect: you know they made Socrates drink hemlock, they owned

slaves, they believed that a bunch of gods partied atop Mount Olympus, they consulted crazed oracles, and they sent poor Hippasus to sleep with the fishes merely because he proved that the square root of 2 could not be expressed as a ratio of integers! But I think that we need to cut them some slack—starting at approximately the time of Hippasus' birth in 500 B.C., Greek civilization exploded, giving the world a huge wealth of impressive art, architecture, literature and new ideas such as democracy and phenomenal progress in areas such as philosophy, mathematics and science. The sudden flowering of Greek civilization is stunning, something at which to marvel after these many centuries, even if that civilization did retain some seriously rough edges.

As it is the case that our group is called the Fellowship of *Reason*, I thought that I would tell a tale today of one type of reasoning, namely, mathematical reasoning. Now some of you may remember the villain in my story, the great mathematician Pythagoras, about whom you learned in geometry class during sophomore year of high school. (Perhaps you might have thought of him as a villain at that time, too!) Specifically, you learned the Pythagorean Theorem, which states that the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of that same triangle. I have here with me a right triangle—that is, a triangle for which one of the internal angles is 90 degrees—and I have denoted the lengths of the legs of this triangle by the symbols a and b , and the hypotenuse has length denoted by c . The Pythagorean Theorem states that $a^2 + b^2 = c^2$, the square of the hypotenuse equals the sum of the squares of the sides.

I have here another right triangle here whose legs are of equal length. For the sake of argument, let us suppose that the length

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of each of these legs is one cubit. (I think that the cubit was the Greek unit of measure; can anyone confirm?) From the Pythagorean Theorem we know that the square of the hypotenuse of this triangle is $1^2 + 1^2 = 1 + 1 = 2$ cubits². Thus, the length of the hypotenuse is the square root of 2 cubits.

Now one can easily take a ruler and measure the hypotenuse and quickly understand that the square root of 2 is some number greater than 1 and less than 2. A bit more exactitude would indicate that the hypotenuse is approximately 7/5 cubits. But that's not quite correct because the square of 7/5 is 49/25 which is still 1/25 shy of equaling 2. The Greeks of Hippasus' time did not use Arabic numbers and decimals—the invention of such a number system laid a millennium in the future. Today I can say to you that the square root of 2 is approximately (but not exactly) 1.414213562373095048801688724209698078569671875...; today I can say that, but an ancient Greek could not have conceptualized it that way for he knew nothing of decimal arithmetic.

The Greeks did understand fractions and they understood that 7/5 was an approximation of the square root of 2, but not a very good one. They would also have been aware that 99/70 was a much better approximation of the square root of 2, but still not exact (the square of 99/70 is 9801/4900, off by only one part in 4900 from exactly equaling 2). They felt sure that there was some exact expression for the square root of 2, but that they simply had not found it yet, and they obsessed over their continued failure to find such a ratio. They were not the first to obsess over this, either; mathematicians had obsessed over this puzzle for over 13 centuries years prior to the Golden Age of Greece. Perhaps as early as 1800 B.C., the Babylonians had inscribed in cuneiform on a clay tablet an algorithm that, using sums and differences of ratios, closely approximated the square root of 2. As a slight digression, tell me: is it amusing, or tragic, or maddening to know that before anyone had ever committed to sheepskin the words "In the beginning, God created the heavens and the earth..."—is it upsetting to know that long before that the square root of 2 was already known

to 6 decimal places? A similar accuracy was achieved on the Indian subcontinent after the Babylonians and before the Greeks.

But even after thirteen centuries of failure to solve the riddle, no one doubted that some as-yet-undiscovered ratio of integers—ratios of integers are called *rational numbers*—would eventually be found such that the square of that ratio would equal 2. 7/5 was not quite right. 1414/1000 was better. 99/70 was better yet. It was absolutely maddening not being able to find the right ratio. The Greeks felt that once the correct fraction had been found, then those two numbers – numerator and denominator both – would have special cosmic significance: at the time, mathematics and mysticism were not well-separated yet.

Hippasus was a disciple of Pythagoras—the men in the boat were known as Pythagoreans—and he was well steeped in the brew of the Pythagorean Theorem and the many so-far unsuccessful attempts to represent the square root of 2 as a ratio. He was also aware of other established mathematical concepts of the time; let me mention a few of these. A prime number is an integer that cannot be divided without remainder by any other integer besides itself and unity; the first few prime numbers are 1, 2, 3, 5, 7, 11, 13, 17, 19, 23 and so forth. The Greeks understood that every integer is either itself prime or else can be factored into a product of prime numbers, a factorization that is unique except for the ordering of those factors. This understanding is now called the Fundamental Theorem of Arithmetic. For example, the number 12 can be expressed as $2 \times 2 \times 3$. 12 can also be written as $3 \times 2 \times 2$, but that is merely a reordering of the factors 2, 2 and 3. 12 can also be written as 4×3 , but 4 is not prime and can itself be expressed as 2×2 . So, no matter what, 12 can only be written as the product of the three prime numbers 2, 2, and 3. As another example, the number 210 can only be written as the product of four prime numbers 2, 3, 5, and 7.

Hippasus' brilliant insight was to assume that the square root of 2 is indeed a rational number, and then through a sequence of correct operations to show that that assumption implies a false result. Because the assumption and the correct operations imply a false result, therefore it is the original assumption that must be false. This type of argument is now referred to as *reductio ad absurdum*, and is one of the standard mathematical proof techniques used today

About the name "The Eudaimonist"

A eudaimonist (pronounced "yoo-DIE-mon-ist") is one who believes that the highest ethical goal is individual happiness and personal well-being. The term is derived from the ancient Greek word eudaimonia, which means, roughly, "well-being."

Disclaimer

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Fellowship of Reason, Inc.

Mission Statement

The Fellowship of Reason® is dedicated to the personal flourishing of our members through reason.

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FOR Merchandise

The mug you have to have.

FOR coffee. FOR tea. FOR a friend.

Mugs are \$6 each or 2 for \$10.



FOR Polo Shirts and T-shirts are also available. Members Price is \$25 for Polo Shirts and \$15 for T-Shirts.

Prices include domestic shipping, inquire for international shipping.

Contact us at any FOR function (see page 9)

Questions? info@fellowshipofreason.com.



Reasoning: Reductio Ad Absurdum (continued from page 2)

in many other contexts.

Let us now consider the classical proof of the irrationality of the square root of 2, proven via reductio ad absurdum.

Theorem: The square root of 2 is irrational.

Proof:

Assume that the square root of 2 is indeed rational, that is, that it can be represented as a ratio p/q , where p and q are two integers. Now if the square root of 2 can be represented as p/q , then we write the equation

$$\sqrt{2} = \frac{p}{q}$$

and square both sides to yield a new equation

$$2 = \frac{p^2}{q^2}$$

Multiplying both sides by q^2 we get another equivalent equation

$$2q^2 = p^2$$

Because of the Fundamental Theorem of Arithmetic mentioned earlier, the p can be represented as the product of, say, k prime numbers: $p = p_1 p_2 \cdots p_k$. For example, if p were $12=2 \times 2 \times 3$, then p would be expressed as the product of $k=3$ factors. For the argument I am making it is not necessary to know what the actual value of k might be; it is some positive integer and that is all we need to know. Because p can be expressed as the product of k factors, therefore p^2 can be expressed as the product of $2k$ factors. For example, 12 can be expressed as the product of three factors $2, 2,$ and 3 and $12^2=144$ can be expressed as the product of six factors $2, 2, 3, 2, 2,$ and 3 . The expression of q^2 also has an even number of terms; supposing that q can be expressed as the product of, say, j prime numbers, then q^2 can be expressed as the product of $2j$ prime numbers. But the equation we face,

$$2q^2 = p^2$$

$$2(q_1 q_2 \cdots q_j)^2 = (p_1 p_2 \cdots)^2$$

must therefore have an even number $2k$ of terms on the right hand side and an odd number $2j+1$ of terms on the left. The left hand side has an odd number of terms because q^2 has an even number $2j$ of terms and the additional 2 multiplier adds one more term to the left hand side, making the number of terms there $2j+1$, which is odd. But this is impossible because the two integers on both sides of the equation are equal and must therefore have the same unique representation as a product of prime numbers in

Reasoning: Reductio Ad Absurdum (continued from page 3)

accordance with the Fundamental Theorem of Arithmetic. The number of terms on both sides of the equation can be either even or odd, but they must agree: the left hand side cannot be odd with the right hand side being even simultaneously. Thus the equation

$$2q^2 = p^2$$

cannot hold. We have made no error in our manipulations and thus the original assumption—the assumption that the square root of 2 is rational—must be false.

The square root of 2 is irrational. Q.E.D.

When a theorem is proven, mathematicians will often write Q.E.D. (quod erat demonstrandum—"which was necessary to be demonstrated") at the end of it. The Q.E.D. is a translation into Latin of a similar phrase in Greek that was used by the great classical Greek mathematicians.

So there we have it. The square root of 2 is irrational. Q.E.D.

Don't throw me overboard!!!

"We have made no error in our manipulations and thus the original assumption—the assumption that the square root of 2 is rational—must be false."

Jerry Pease performs with Atlanta Philharmonic Orchestra, Saturday, August 11 at 7:30 p.m.

Come and support FOR member Jerry Pease and his new musical adventure with the Atlanta Philharmonic Orchestra.

www.atlantaphilharmonic.org

Performing:

Elgar: Enigma Variations
Beethoven: Symphony No. 5
Mascagni: Intermezzo from Cavalleria Rusticana

Summer Concert, Saturday, August 11, 2007, 7:30 p.m. at the North Decatur Presbyterian Church, 611 Medlock Road, Decatur.

Meaning of Membership

Regular membership:
\$35 per year

Contributing membership:
\$70 per Year

Lifetime membership:
\$700

In order to enjoy the benefits of the Fellowship of Reason, you need not be a member. All of our activities are free and open to the public. A number of non-members regularly participate with us and are most heartily welcome.

Why then become a member of the Fellowship of Reason?

The Fellowship of Reason exists by virtue of the generosity of its members and friends. People you know contribute money, property, services, and time in order that the value we all enjoy continues to exist. The services by performers at FORum, by executive committee members, by hosts and hostesses, and by planners of events

are vital to the existence of FOR. Those volunteer services are, in fact, FOR.

Membership is, simply, a clear statement that you wish the Fellowship of Reason to continue to exist. Membership is not the only way to make this statement. Some non-members are generous contributors of money, property, and services. In fact, we all benefit by the mere attendance of non-members at FOR events.

If you want FOR to continue to exist, why not clearly say so by becoming a member today?

Classic Film Appreciation Course

Classic Film Appreciation Course
By Steve Whiteman

Throughout 2007 FOR member **Steve Whiteman** will teach a free course in the appreciation of classic film and associated early and mid-20th century popular culture. The course is full, but seats may become available to see individual films.

For more information, or to receive e-mail, contact Steve at classic.film@yahoo.com

The movies will be acknowledged classics, the majority from Hollywood's Golden Age—the 1930s and 1940s—and the rest from other eras or other countries. The program usually will include a cartoon or a short subject, coming attractions, the feature film, and commentary on the careers of key personnel, filmmaking techniques, and relevant history and popular culture. Discussion will follow the commentary.

First and Second Saturdays in 2007
Doors open 6:45 p.m. program starts at
7:30 p.m.

Contact Information

To talk to somebody about Fellowship of Reason call:

Martin at 770-471-9800

Dan at 770-831-3010

Change of Address

By Kathleen Allen

FOR IMMEDIATE RELEASE

Kathleen Hamilton Allen publishes debut novel *Change of Address*. Readers will cry before they laugh. It may anger them, but only because it makes them think about the factors that influence their own lives.

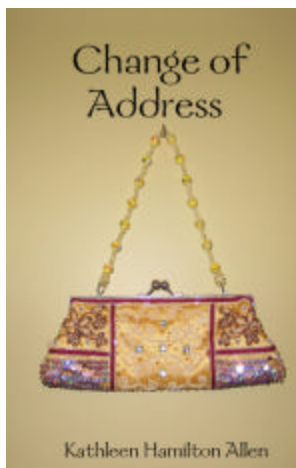
NORCROSS, GA — JULY 17, 2007 — *Change of Address* takes an incisive look at the factors we rely upon for survival and spiritual sustenance – Faith, Family and Friends, and the impact upon them of Fame and Fortune, often desired in theory, yet rarely enjoyed in reality.

Kathleen Hamilton Allen wrote *Change of Address* after she and her husband moved a continent away from family and friends to settle in a small-town, Bible-belt community. A summer without jobs or TV meant time tending two small children and only internet news for entertainment, with ample time to question the impact of the above factors on our lives.

Within her tale of a widowed mother's loss and recovery, the author casts a critical eye on:

- forces within contemporary, conservative churches;
- de-facto segregation in early-childhood education;
- the public's insatiable desire for scandal, however inconsequential the story; and
- whether money really does buy happiness.

Her tale is ultimately hopeful, but will spark passionate responses from her readers.



Change of Address is available in paperback (\$20.00) and e-book (\$10.00) formats.

Link to Publication:
<http://www.lulu.com/content/980983>

ABOUT AUTHOR

Kathleen Hamilton Allen lives with her husband and two children in a suburb of Atlanta. She graduated from Claremont McKenna College, Claremont, CA with a dual major of Government/French.

FOR's mission is to promote the personal flourishing of our members through reason.

August 2007

August 2007							September 2007						
S	M	T	W	T	F	S	S	M	T	W	T	F	S
5	6	7	8	9	10	11	2	3	4	5	6	7	8
12	13	14	15	16	17	18	9	10	11	12	13	14	15
19	20	21	22	23	24	25	16	17	18	19	20	21	22
26	27	28	29	30	31		23	24	25	26	27	28	29
							30						

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			August 1	2	3	4
			Beth bd 7:30pm Invisible College (Greg's house)			7:00pm Classic Film Appreciation Course (Steve Whiteman's house)
5	6	7	8	9	10	11
12:00pm FORum (NWUUC)		7:30pm Philosophy Tapes (Sally's house)	Dave D. bd 7:30pm Invisible College (Greg's house)			11:00am Nonfiction Book Club (Allison's house) 7:00pm Classic Film Appreciation 7:30pm Jerry Pease (Atlanta Phil)
12	13	14	15	16	17	18
8:00am FOR Runners (Flying Biscuit)		7:30pm Fiction Book Club (Beth's house)	7:30pm Invisible College (Greg's house)			Carlos bd Dunham bd 5:00pm Potluck kid-friendly (Ron/Susan Menich)
19	20	21	22	23	24	25
8:00am FOR Runners (Flying Biscuit)		7:30pm Philosophy Tapes (Sally's house)	7:30pm Invisible College (Greg's house)	8:00pm FOR pub	Jabbo bd	Holly bd 6:30pm Movie Night (John's studio)
26	27	28	29	30	31	
8:00am FOR Runners (Flying Biscuit)		Tom bd	7:30pm Invisible College (Greg's house)		Nick bd	

September 2007

September 2007							October 2007						
S	M	T	W	T	F	S	S	M	T	W	T	F	S
2	3	4	5	6	7	8	7	8	9	10	11	12	13
9	10	11	12	13	14	15	14	15	16	17	18	19	20
16	17	18	19	20	21	22	21	22	23	24	25	26	27
23	24	25	26	27	28	29	28	29	30	31			
30													

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
						September 1 7:00pm Classic Film Appreciation Course (Steve Whiteman's house)
2 12:00pm FORum (NWUUC)	3	4 7:30pm Philosophy Tapes (Sally's house)	5 7:30pm Invisible College (Greg's house)	6	7	8 11:00am Nonfiction Book Club (Allison's house) 7:00pm Classic Film Appreciation Course (Steve Whiteman's house)
9 8:00am FOR Runners (Flying Biscuit)	10	11 7:30pm Fiction Book Club (Beth's house)	12 7:30pm Invisible College (Greg's house)	13	14	15 7:30pm Potluck (Volunteer)
16 8:00am FOR Runners (Flying Biscuit)	17	18 7:30pm Philosophy Tapes (Sally's house)	19 Jim bd 7:30pm Invisible College (Greg's house)	20	21	22 6:30pm Movie Night (John's studio)
23 8:00am FOR Runners (Flying Biscuit)	24	25	26 7:30pm Invisible College (Greg's house)	27 7:00pm Atlanta Symphony (La Boheme) 8:00pm FOR pub	28	29
30						

Events

For detailed info on all upcoming events, visit
<http://www.fellowshipofreason.com/calendar.htm>

FORum: A Celebration of Human Achievement

First Sunday noon
 FOR's premier event. Starting at noon with an InFORmation Hour conducted by Dan Barber for visitors and new members. Meet and greet at 12:30 p.m. The program starts at 1 p.m. and lasts one hour. We enjoy post-program conversation until 3 p.m. when those interested adjourn to The Loop restaurant for further fellowship. Children's Program from noon to 3 p.m.

Martin Cowen director: 770-471-9800.

FOR Runners: Sunday 8 a.m.

We meet every Sunday morning at 8 a.m., except FORum Sundays, at the southwest corner of Piedmont Park on 10th Street. Breakfast at Flying Biscuit (corner of Piedmont and 10th Street) follows at 9 a.m. Breakfast lovers, walkers, and joggers welcome!

Martin Cowen coordinator: 770-471-9800.

Philosophy Club: 1st/ 3rd Tuesdays 7:30 p.m.

Philosophy tapes are played to a small group of friends in a private home on the 1st and 3rd Tuesdays of each month. Free.

Sally Hull coordinator: 404-257-0454

Fiction Book Club: 2nd Tuesday 7:30 p.m.

Members and friends of FOR meet on the 2nd Tuesday of every month at John and Beth's house. For book selections and more information, go to www.fellowshipofreason.com/fiction.htm

Beth Holley coordinator: 404-372-7378

Non-Fiction Book Club: 2nd Saturday 11 a.m.

Members and friends of FOR meet 11 a.m. on the 2nd Saturday of every month at Chris and Allison's house. For book selections and more information go to:

www.fellowshipofreason.com/nonfiction.htm

Allison Byrd coordinator: 404-372-4089

Kid-Friendly Potluck: 3rd Saturday, 5:00 p.m.

Our next kid-friendly Potluck is Saturday, August 18, 2007 at 5 p.m. Ron and Susan Menich's house. Remember, it is potluck, so bring something good to eat and drink to share.

Pub Night: 4th Thursday, 8 p.m.

Members and friends of FOR meet on the 4th Thursday of every month at Manuel's Tavern at 8 p.m. for adult beverages and adult conversation. FOR's next pub night is Thursday, Thursday, August 23, 2007 at 8 p.m.

Scott Carper coordinator: 404-964-6697

Movie Night: 4th Saturday, 6:30 p.m.

Members and friends of FOR meet on the 4th Saturday of every month at John Grover's photography studio to enjoy potluck and a movie. FOR's next movie night is Saturday, August 25, 2007, at 6:30 p.m.

John Grover coordinator: 404-872-4555

*Join us for our
 next monthly
 FORum:*

August 5, 2007
Sunday at 1 p.m.
 (Meet, Greet at 12:30)

Come early for the
**InFORmation
 Hour**

At noon
 Moderated by Dan Barber for
 visitors and new members

And NEW THIS MONTH
FOR FAMILY FRIENDLY
 Children's program
 from noon to 3 p.m.

Birthdays

- | | |
|-------------|--------|
| • August 1 | Beth |
| • August 18 | Carlos |
| • August 18 | Dunham |
| • August 24 | Edward |
| • August 25 | Holly |
| • August 28 | Tom |
| • August 31 | Nick |

- | | |
|----------------|-----|
| • September 19 | Jim |
|----------------|-----|

Celebratory Announcements

Do yourself a favor and remember a good thing that happened to you this month:

Please, write it down: _____

Now do the membership of FOR, Inc. a favor by relating this fact during FORum next month!



Fellowship of Reason

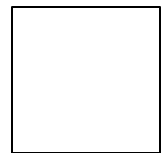
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Community for the 21st
Century

FELLOWSHIP OF REASON, INC.
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Atlanta, Georgia 31107



We're on the Web!
See us at:

fellowshipofreason.com